fourth GRADE

2018-2019 Guide

January 14th - March 22nd

Eureka

Module 5: *Fraction Equivalence, Ordering and Operations*

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

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Module 5 Performance Overview

- Topic A builds on Grade 3 work with unit fractions. Students explore fraction equivalence through the decomposition of non-unit fractions into unit fractions (fractions where the numerator is one and the denominator is some other number), as well as the decomposition of unit fractions into smaller unit fractions. They represent these decompositions, and prove equivalence, using visual models.
- In Topic B, students begin generalizing their work with fraction equivalence. Students learn that fractions can be expressed with larger or smaller units (eg. 3 / 4 = 9 / 12). They see that any unit fraction length can be partitioned into n equal lengths. For example, each third in the interval from 0 to 1 may be partitioned into 4 equal parts. Doing so multiplies both the total number of fractional units (the denominator) and the number of selected units (the numerator) by 4. Conversely, students see that, for example, when the interval from 0 to 1 is partitioned into twelfths, one may group 4 twelfths at a time to make thirds. By doing so, both the total number of fractional units and number of selected units are divided by 4.
- Students use the relationship between the numerator and denominator of a fraction to compare given fractions. For example, when comparing 4/7 and 2/5 , students reason that 4 sevenths is more than 1 half, while 2 fifths is less than 1 half. They then conclude that 4 sevenths is greater than 2 fifths. They finish by understanding the best method to compare fractions.
- Topic D bridges students' understanding of whole number addition and subtraction to fractions. Everything that they know to be true of addition and subtraction with whole numbers now applies to fractions. Addition is finding a total by combining like units. Subtraction is finding an unknown part.
- In Topic E, students study equivalence involving both ones and fractional units. They use decomposition and visual models to add and subtract fractions less than 1 to and from whole numbers. Students use addition and multiplication to build fractions greater than 1 and then represent them on the number line.
- Topic F provides students with the opportunity to use their understandings of fraction addition and subtraction as they explore mixed number addition and subtraction by decomposition. Students once again call on their understanding of benchmark fractions as they determine, prior to performing the actual operation, what a reasonable outcome will be.
- Topic G extends the concept of representing repeated addition as multiplication, applying this familiar concept to work with fractions. Multiplying a whole number times a fraction was introduced in Topic A as students learned to decompose fractions.
- Topic H is an exploration lesson in which students find the sum of all like denominators from 0/n to n/n . Students first work, in teams, with fourths, sixths, eighths, and tenths.

Module 5: *Fraction Equivalence, Ordering, and Operations*

Pacing: January 14th - March 22nd **44 Days**		
Topic	**Lesson**	**Lesson Objective/ Supportive Videos**
Topic A: Decomposition and Fractions Equivalence	Lesson 1	Decompose fractions as a sum of unit fractions using tape diagrams. https://www.youtube.com/watch?v
	Lesson 2	Decompose fractions as a sum of unit fractions using tape diagrams. https://www.youtube.com/watch?v
	Lesson 3	Decompose non-unit fractions and represent them as a whole number times a unit fraction using tape diagrams. https://www.youtube.com/watch?v
	Lesson 4	Decompose fractions into sums of smaller unit fractions using tape diagrams. https://www.youtube.com/watch?v
	Lesson 5	Decompose unit fractions using area models to show equivalence. https://www.youtube.com/watch?v
	Lesson 6	Decompose fractions using area models to show equivalence. https://www.youtube.com/watch?v
Topic B: Fraction Equivalence using Multiplication and Division	Lesson 7	Use the area model and multiplication to show the equivalence of two fractions. https://www.youtube.com/watch?v
	Lesson 8	Use the area model and multiplication to show the equivalence of two fractions. https://www.youtube.com/watch?v
	Lesson 9	Use the area model and division to show the equivalence of two fractions. https://www.youtube.com/watch?v
	Lesson 10	Use the area model and division to show the equivalence of two fractions. https://www.youtube.com/watch?v
	Lesson 11	Explain fraction equivalence using a tape diagram and the number line, and relate that to the use of multiplication and division. https://www.youtube.com/watch?v
Topic C:	Lesson	Reason using benchmarks to compare two fractions on the

Fraction Comparison	12	number line. https://www.youtube.com/watch?v
	Lesson 13	Reason using benchmarks to compare two fractions on the number line. https://www.youtube.com/watch?v
	Lesson 14	Find common units or number of units to compare two fractions. https://www.youtube.com/watch?v
	Lesson 15	Find common units or number of units to compare two fractions. https://www.youtube.com/watch?v
Topic D: Fraction Addition and Subtraction	Lesson 16	Use visual models to add and subtract two fractions with the same units. https://www.youtube.com/watch?v
	Lesson 17	Use visual models to add and subtract two fractions with the same units, including subtracting from one whole. https://www.youtube.com/watch?v
	Lesson 18	Add and subtract more than two fractions. https://www.youtube.com/watch?v
	Lesson 19	Solve word problems involving addition and subtraction of fractions. https://www.youtube.com/watch?v
	Lesson 20	Use visual models to add two fractions with related units using the denominators 2, 3, 4, 5, 6, 8, 10, and 12. https://www.youtube.com/watch?v
	Lesson 21	Use visual models to add two fractions with related units using the denominators 2, 3, 4, 5, 6, 8, 10, and 12. https://www.youtube.com/watch?v
Mid Module Assessment February 14-15, 2019		
Topic E: Extending Fraction Equivalence to Fractions Greater than 1	Lesson 22	Add a fraction less than 1 to, or subtract a fraction less than 1 from, a whole number using decomposition and visual models. https://www.youtube.com/watch?v
	Lesson 23	Add and multiply unit fractions to build fractions greater than 1 using visual models. https://www.youtube.com/watch?v
	Lesson 24	Decompose and compose fractions greater than 1 to express them in various forms. https://www.youtube.com/watch?v

	Lesson 25	Decompose and compose fractions greater than 1 to express them in various forms. https://www.youtube.com/watch?v
	Lesson 26	Compare fractions greater than 1 by reasoning using benchmark fractions. https://www.youtube.com/watch?v
	Lesson 27	Compare fractions greater than 1 by creating common numerators or denominators. https://www.youtube.com/watch?v
	Lesson 28	Solve word problems with line plots. https://www.youtube.com/watch?v
Topic F: Addition and Subtraction of Fractions by Decomposition	Lesson 30	Add a mixed number and a fraction. https://www.youtube.com/watch?v
	Lesson 31	Add mixed numbers. https://www.youtube.com/watch?v
	Lesson 32	Subtract a fraction from a mixed number. https://www.youtube.com/watch?v
	Lesson 33	Subtract a mixed number from a mixed number. https://www.youtube.com/watch?v
	Lesson 34	Subtract mixed numbers. https://www.youtube.com/watch?v
Topic G: Repeated Addition of Fractions as Multiplication	Lesson 35	Represent the multiplication of n times a/b as (n × a)/b using the associative property and visual models. https://www.youtube.com/watch?v
	Lesson 36	Represent the multiplication of n times a/b as (n × a)/b using the associative property and visual models. https://www.youtube.com/watch?v
	Lesson 37	Find the product of a whole number and a mixed number using the distributive property. https://www.youtube.com/watch?v
	Lesson 38	Find the product of a whole number and a mixed number using the distributive property. https://www.youtube.com/watch?v
	Lesson 39	Solve multiplicative comparison word problems involving fractions. https://www.youtube.com/watch?v
	Lesson 40	Solve word problems involving the multiplication of a whole number and a fraction including those involving line plots. https://www.youtube.com/watch?v
Topic H: Exploring a Fraction Pattern	Lesson 41	Find and use a pattern to calculate the sum of all fractional parts between 0 and 1. Share and critique peer strategies. https://www.youtube.com/watch?v
End of Module Assessment **March 21-22, 2019**		

NJSLS Standards:

4.OA.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

- Patterns involving numbers or symbols either repeat or grow. Students need multiple opportunities creating and extending number and shape patterns. Numerical patterns allow students to reinforce facts and develop fluency with operations. Create rules that patterns follow by using a t-chart or visual representation

- Patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will look like. Students investigate different patterns to find rules, identify features in the patterns, and justify the reason for those features.

- Investigate different patterns to find rules, identify features in the patterns, and justify the reason for those features. Describe features of an arithmetic number pattern or shape pattern by identifying the rule and features that are not explicit in the rule.

Pattern	Rule	Feature(s)
3, 8, 13, 18, 23, 28…	Start with 3, add 5	The numbers alternately end with a 3 or 8
5, 10, 15, 20…	Start with 5, add 5	The numbers are multiples of 5 and end with either 0 or 5. The numbers that end with 5 are products of 5 and an odd number. The numbers that end in 0 are products of 5 and an even number.

Example:
Rule: Starting at 1, create a pattern that starts at 1 and multiplies each number by 3. Stop when you have 6 numbers.
Students write 1, 3, 9, 27, 81, 243. Students notice that all the numbers are odd and that the sums of the digits of the 2 digit numbers are each 9. Some students might investigate this beyond 6 numbers.

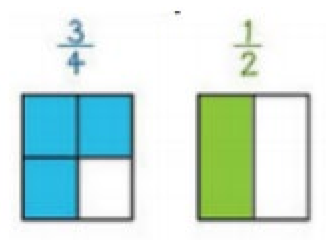
- Another feature to investigate is the patterns in the differences of the numbers (3 - 1 = 2, 9 - 3 = 6, 27 - 9 = 18, This standard calls for students to describe features of an arithmetic number pattern or shape pattern by identifying the rule, and features that are not explicit in the rule.

4.NF.1	Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. [Grade 4 expectations in this domain are limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100.]

- This standard refers to visual fraction models. This includes area models, number lines or it could be a collection/set model. This standard extends the work in third grade by using additional denominators. (5, 10, 12, and 100)

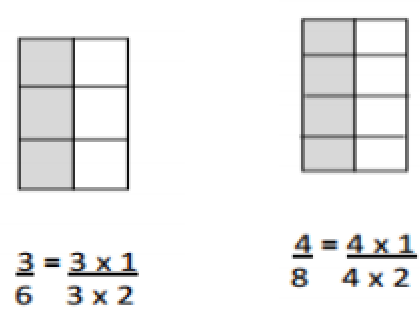
- This standard addresses equivalent fractions by examining the idea that equivalent fractions can be created by multiplying both the numerator and denominator by the same number or by dividing a shaded region into various parts.

- Create visual fraction models using benchmark fractions to compare and determine if fractional parts are equivalent.



- Visual fraction models include the use of area models, number lines, or a collection/set model and can be used to explain why fractions are equivalent. Models of fractions can be used to generate a rule for writing equivalent fractions.

- Equivalent fractions are the same size while the number and size of the parts differ. Generate equivalent fractions, using fraction a/b as equivalent to fraction (n x a) (n x b)



- Students should begin to notice connections between the models and fractions in the way both the parts and wholes are counted and begin to generate a rule for writing equivalent fractions.

- There is **NO** mathematical reason why fractions must be written in simplified form, although it may be convenient to do so in some cases.

<table>
<tr><td>4.NF.2</td><td>Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
[Grade 4 expectations in this domain are limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100.]</td></tr>
</table>

- Visual concrete fractional models are an important initial component in developing the conceptual understanding of fractions.
- Visual fraction models or finding common denominators or numerators can be used to compare fractions. Identify and give multiple representations for the fractional parts of a whole (area model) or set.
- Utilize area models, number lines, double number lines, verbal justification, and benchmark fractions to compare fractional parts.
- **Fractions may only be compared when the two fractions are referring to the same whole**.

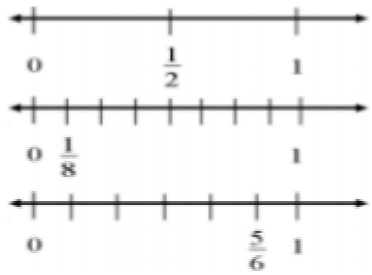
Fractions with common denominators may be compared using the numerators as a guide

$$\frac{2}{6} < \frac{3}{6} < \frac{5}{6}$$

- Fractions with common numerators may be compared and ordered using the denominators as a guide.

$$\frac{3}{10} < \frac{3}{8} < \frac{3}{4}$$

- Fractions may be compared using $\frac{1}{2}$ as a benchmark.



- Record the results of comparisons with the symbol >, =, or < and justify the have the same numerator, the fraction with the lesser denominator is the greater fraction.
- Fractions with the same size pieces, or common denominators, within the same size whole can be compared with each other because the size of the pieces is the same. Comparison to known benchmark quantities can help determine the relative size of fractional pieces because the benchmark quantity can be seen as greater, less than, or the same as the piece.

4.NF.3	Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.
4.NF.B.3.A	Understand addition and subtraction of fractions as joining and separating parts referring to the same whole
4.NF.B.3.B	Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
4.NF.B.3.C	Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
4.NF.B.3.D	Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. [Grade 4 expectations in this domain are limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100.]

- A fraction with a numerator of one is called a unit fraction. When students investigate fractions other than unit fractions, such as $\frac{2}{3}$, they should be able to join (compose) or separate (decompose) the fractions of the same whole. Create equations that identify the decomposing (iterating) of fractions. Being able to visualize this decomposition into unit fractions helps students when adding or subtracting fractions.

Example:

$\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$

- Students need multiple opportunities to work with mixed numbers and be able to decompose them in more than one way. Students may use visual models to help develop this understanding.

Example of word problem:

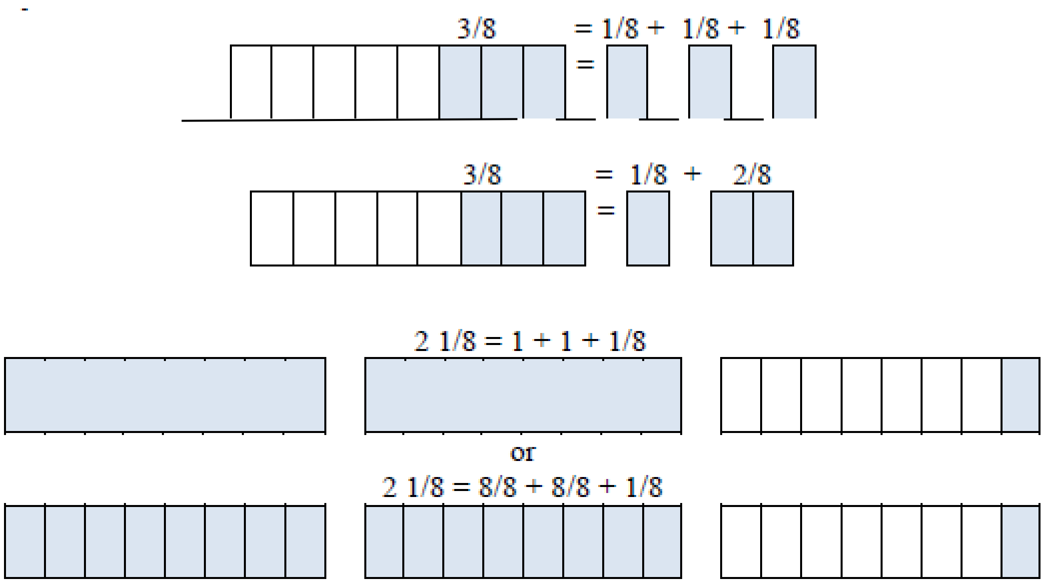
Mary and Lacey decide to share a pizza. Mary ate 3/6 and Lacey ate 2/6 of the pizza. How much of the pizza did
the girls eat together?

Possible solution:

The amount of pizza Mary ate can be thought of as $\frac{3}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$. The amount of pizza Lacey ate can be thought of as $\frac{1}{6}$ and $\frac{1}{6}$. The total amount of pizza they ate is $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ or $\frac{5}{6}$ of the whole.

- Students should justify their breaking apart (decomposing) of fractions using visual fraction models. A mixed number is a whole number and a fractional part which can be written as a fraction with a numerator greater than the denominator. The concept of turning mixed numbers into improper fractions needs to be emphasized using visual fraction models.

  Pizza Example:



- Converting an improper fraction to a mixed number is a matter of decomposing the fraction into a sum of a whole number and a number less than 1. Students can draw on their knowledge from third grade of whole numbers as fractions.

Example,
Knowing that 1 = 3/3, they see:

$$\frac{5}{3} = \frac{3}{3} + \frac{2}{3} = 1 + \frac{2}{3} = 1\frac{2}{3}$$

- A separate algorithm for mixed numbers in addition and subtraction is not necessary. Students will tend to add or subtract the whole numbers first and then work with the fractions using the same strategies they have applied to problems that contained only fractions.

Example:
Susan and Maria need 8 3/8 feet of ribbon to package gift baskets. Susan has 3 1/8 feet of ribbon and Maria has 5 3/8 feet of ribbon. How much ribbon do they have altogether? Will it be enough to complete the project? Explain why or why not.

The student thinks:
I can add the ribbon Susan has to the ribbon Maria has to find out how much ribbon they have altogether. Susan has 3 1/8 feet of ribbon and Maria has 5 3/8 feet of ribbon. I can write this as 3 1/8 + 5 3/8. I know they have 8 feet of ribbon by adding the 3 and 5. They also have 1/8 and 3/8 which makes a total of 4/8 more.
Altogether, they have 8 4/8 feet of ribbon. 8 4/8 is larger than 8 3/8 so they will have enough ribbon to complete the project. They will even have a little extra ribbon left, 1/8 foot.

Example:
Trevor has $4\frac{1}{8}$ pizzas left over from his soccer party. After giving some pizza to his friend, he has $2\frac{4}{8}$ of a pizza left. How much pizza did Trevor give to his friend?

Possible solution:
Trevor had $4\frac{1}{8}$ pizzas to start. This is $\frac{33}{8}$ of a pizza. The x's show the pizza he has left which is $2\frac{4}{8}$ pizzas or $\frac{20}{8}$ pizzas. The shaded rectangles without the x's are the pizza he gave to his friend which is $\frac{13}{8}$ or $1\frac{5}{8}$ pizzas.

- Mixed numbers are introduced for the first time in Fourth Grade. Students should have ample experiences of adding and subtracting mixed numbers where they work with mixed numbers or convert mixed numbers so that the numerator is equal to or greater than the denominator.

- Fourth Grade students should be able to decompose and compose fractions with the same denominator.

- Using the understanding gained from work with whole numbers of the relationship between addition and subtraction, they also subtract fractions with the same denominator. For example, to subtract $\frac{5}{6}$ from $\frac{17}{6}$, they decompose.

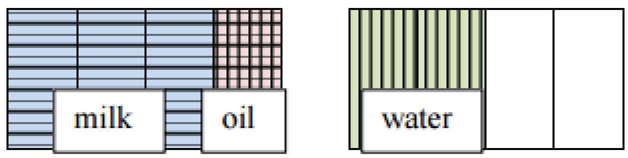
$$\frac{12}{6}+\frac{5}{6}, \quad \text{so} \quad \frac{17}{6}-\frac{5}{6}=\frac{17-5}{6}=\frac{12}{6}=2.$$

- Students also compute sums of whole numbers and fractions, by representing the whole number as an equivalent fraction with the same denominator as the fraction. Example:

$$7\frac{1}{5}=7+\frac{1}{5}=\frac{35}{5}+\frac{1}{5}=\frac{36}{5}.$$

- Students use this method to add mixed numbers with like denominators.
- ***Converting a mixed number to a fraction should not be viewed as a separate technique to be learned by rote, but simply as a case of fraction addition.***

A cake recipe calls for you to use $\frac{3}{4}$ cup of milk, $\frac{1}{4}$ cup of oil, and $\frac{2}{4}$ cup of water. How much liquid was needed to make the cake?



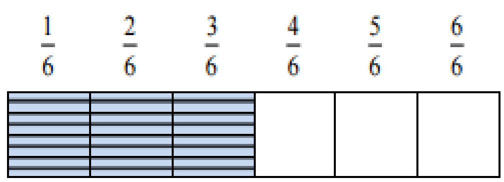
$$\frac{3}{4} \quad + \quad \frac{1}{4} \quad + \quad \frac{2}{4} \quad = \quad \frac{6}{4} \quad or\ 1\,\frac{2}{4}$$

- Addition and subtraction can be divided into four categories that can be represented by different models: joining action, separating action, part-part-whole relations, and comparing situations.
- Emphasize the misconception of combining both the numerators and the denominators for the total fractional amount. For example: $\frac{11}{4}+\frac{7}{4}=\frac{18}{8}$.
- Practice counting with fractions, such as skip counting, counting on, and counting back. This will help students see patterns and develop proficiencies that will strengthen their understanding of addition and subtraction. Try asking students to count by halves starting with 0 or count by thirds starting at 10.

4.NF.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
4.NF.B.4.A	Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
4.NF.B.4.B	Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
.NF.B.4.C	Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

- Present opportunities for students to see that multiplying whole numbers by fractions results in a smaller product. This is opposite to a previous idea learned; the operation of multiplication produces a larger product.
- Another way to read the multiplication symbol is "of".
- Improper fractions can be shown visually to show relationships between numerator and denominator.

Area Model

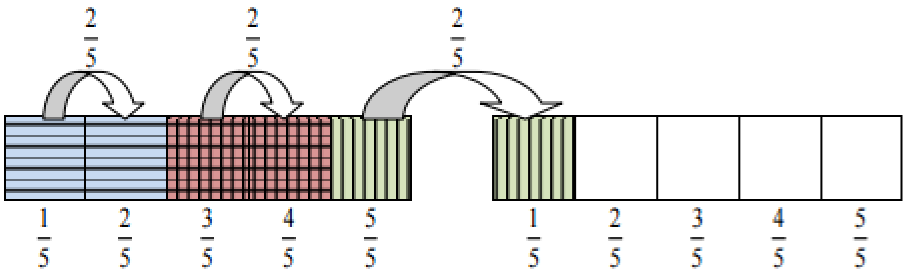


Students should see a fraction as the numerator times the unit fraction with the same denominator.

Example:

$$\frac{7}{5} = 7 \times \frac{1}{5}, \qquad \frac{11}{3} = 11 \times \frac{1}{3}.$$

- This standard extended the idea of multiplication as repeated addition. For example, 3 x ($\frac{2}{5}$) = $\frac{2}{5}$ +$\frac{2}{5}$ +$\frac{2}{5}$ =$\frac{6}{5}$ = 6 x ($\frac{1}{5}$). Students are expected to use and create visual fraction models to multiply a whole number by a fraction.



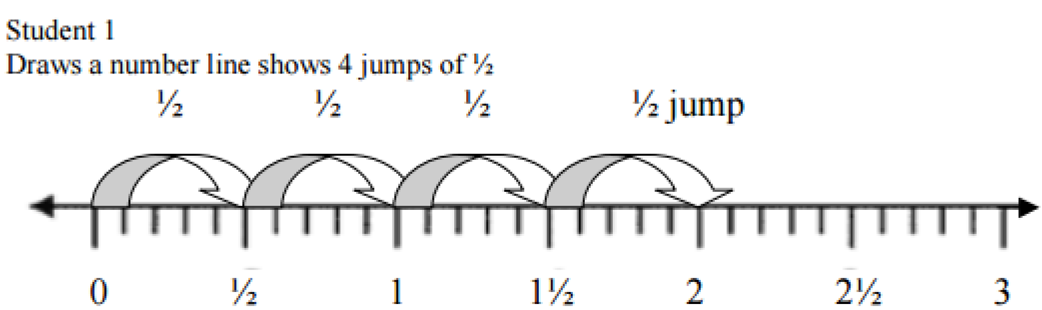
- The same thinking, based on the analogy between fractions and whole numbers, allows students to give meaning to the product of whole number and a fraction.

Example:

$$3 \times \frac{2}{5} \quad \text{as} \quad \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{3 \times 2}{5} = \frac{6}{5}.$$

- When introducing this standard make sure student use visual fraction models to solve word problems related to multiplying a whole number by a fraction.

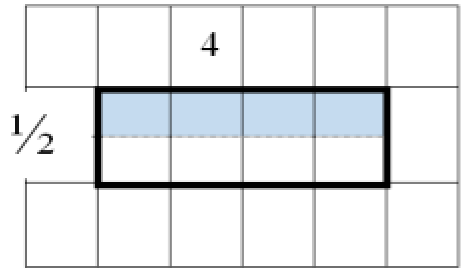
Example:
In a relay race, each runner runs ½ of a lap. If there are 4 team members how long is the race?



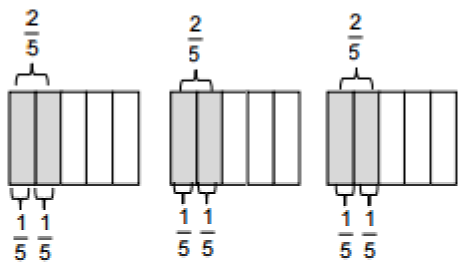
Student 2

Draws an area model showing 4 pieces of ½ joined together to equal 2.

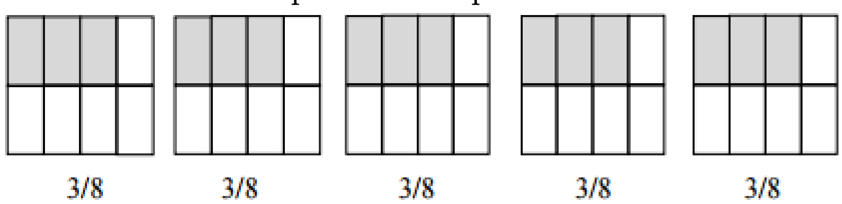
Student 3
Draws an area model representing 4 x ½ on a grid, dividing one row into ½ to represent the multiplier

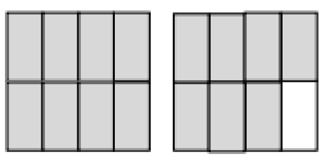


Example:
Students need many opportunities to work with problems in context to understand the connections between models and corresponding equations. Contexts involving a whole number times a fraction lend themselves to modeling and examining patterns.
Examples: $3 \times (\frac{2}{5}) = 6 \times (\frac{1}{5}) = \frac{6}{5}$



If each person at a party eats $\frac{3}{8}$ of a pound of roast beef, and there are 5 people at the party, how many pounds of roast beef are needed? Between what two whole numbers does your answer lie? A student may build a fraction model to represent this problem:





3/8 + 3/8 + 3/8 + 3/8 + 3/8 = 15/8 = 1 7/8

- Students solve word problems involving multiplication of a fraction by a whole number.

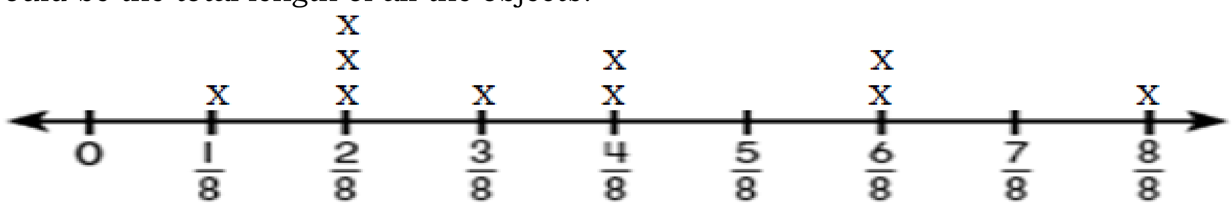
Example: If a bucket holds $2\frac{3}{4}$ gallons and 43 buckets of water fill a tank, how much does the tank hold? The solution $43 \times 2\frac{3}{4}$ gallons, one possible way to solve problem.

$$43 \times \left(2 + \frac{3}{4}\right) = 43 \times \frac{11}{4} = \frac{473}{4} = 118\frac{1}{4} \text{ gallons}$$

4.MD.4	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

- This standard provides a context for students to work with fractions by measuring objects to an eighth of an inch. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot. Rulers can be used to create line plots and review equivalent fractions using eights, fourths, and halves.

Example:
Students measured objects in their desk to the nearest 1/2, 1/4 or1/8 inch. They displayed their data collected on a line plot. How many objects measured an inch? If you put all the objects together end to end what
would be the total length of all the objects?



- Given a set of data, create a graph, describe a context for the data, explain a possible collection method and report what was learned from the data.
- When counting fractional parts on a number line, use the fraction name instead of the whole-number name. For example, if two-fourths is represented on the line plot three times, then there would be six fourths.
- Solve problems involving addition and subtraction of fractions with like denominators by using data presented in the line plots.
- Challenge students to reason using appropriate mathematical language while interpreting data on a line plot.
- Develop a clear understanding for the need to label line plots appropriately. Given a data set consisting of measurements in fractions of a unit, create a line plot.
- **The scale of a line plot must be equally spaced as in a number line.**

Common multiplication and division situations. [1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	**3 x ? = 18**, and **18 ÷ 3 = ?**	**? x 6 = 18**, and **18 ÷ 6 = ?**
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS [2], AREA [3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	**a x b = ?**	**a x ? = p** and **p ÷ a = ?**	**? x b = p**, and **p ÷ b = ?**

[1] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Module 5 Assessment / Authentic Assessment Recommended Framework			
Assessment	**CCSS**	**Estimated Time**	**Format**
Eureka Math Module 5: Fraction Equivalence, Ordering, and Operations			
Authentic Assessment	4.NF.3	30 mins	Individual
Optional Mid-Module Assessment	4.NF.1 4.NF.2 4.NF.3 4.NF.4	1 Block	Individual
Optional End of Module Assessment	4.OA.5 4.NF.1 4.NF.2 4.NF.3 4.NF.4 4.MD.4	1 Block	Individual

Fourth Grade Ideal Math Block

Getting Ready: Whole Group
Number Talk/ Number Strings

7-10 min.

Fluency: Whole Group
Sprints, Counting, Whiteboard Exchange

Application Problem: Whole Group
Provides HANDS-ON work to allow children to ACT OUT or ENGAGE ACTIVELY with the new MATH IDEA

Concept Development: Individual/partner/whole
Instruction & Strategic Problem Set Questions

Student Debrief: Independent
Exit Ticket

50-60 min.

CENTERS/STATIONS:
Pairs / Small Group/ Individual
DIFFERENTIATED activities designed to **RETEACH, REMEDIATE, ENRICH** student's understanding of concepts.

M: Meet with the teacher	A: Application/ Problem Solving	T: Technology	H: Hands on Activities

15-20 min.

SUMMARY: Whole Group
Lesson Closure: Student Reflection; Real Life Connections to Concept

Up to 5 min.

Eureka Lesson Structure:

Fluency:

- Sprints
- Counting : Can start at numbers other than 0 or 1 and might include supportive concrete material or visual models
- Whiteboard Exchange

Application Problem:

- Engage students in using the RDW Process
- Sequence problems from simple to complex and adjust based on students' responses
- Facilitate share and critique of various explanations, representations, and/or examples.

Concept Development: (largest chunk of time)

Instruction:

- Maintain overall alignment with the objectives and suggested pacing and structure.
- Use of tools, precise mathematical language, and/or models
- Balance teacher talk with opportunities for peer share and/or collaboration
- Generate next steps by watching and listening for understanding

Problem Set: (Individual, partner, or group)

- Allow for independent practice and productive struggle
- Assign problems strategically to differentiate practice as needed
- Create and assign remedial sequences as needed

Student Debrief:

- Elicit students thinking, prompt reflection, and promote metacognition through student centered discussion
- Culminate with students' verbal articulation of their learning for the day
- Close with completion of the daily Exit Ticket (opportunity for informal assessment that guides effective preparation of subsequent lessons) as needed.

PARCC Assessment Evidence/Clarification Statements			
CCSS	**Evidence Statement**	**Clarification**	**MP**
4.OA.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.	• Tasks do not require students to determine a rule; the rule is given. • 75% of patterns should be number patterns	MP 8
4.NF.1-2	Use the principle a/b = (nxa)/(nxb) to recognize and generate equivalent fractions.	• The explanation aspect of 4.NF.1 is not assessed here. • Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. • Tasks may include fractions that equal whole numbers. Whole numbers are limited to 0 through 5.	MP 7
4.NF.2-1	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or by comparing to a benchmark fraction such as 1/2. Record the results of comparisons with symbols >, =, or <.	• Only the answer is required. • Tasks require the student to choose the comparison strategy autonomously. • Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. • Tasks may include fractions that equal whole numbers. Whole numbers are limited to 0 through 5	MP 6,7
4.NF.A.Int.1	Apply conceptual understanding of fraction equivalence and ordering to solve simple word problems requiring fraction comparison. Content Scope: 4.NF.A	• Tasks have "thin context." • Tasks do not require adding, subtracting, multiplying, or dividing fractions. • Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy. • Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. • Tasks may include fractions that equal whole numbers. Whole numbers are limited to 0 through 5.	MP 1,4,5

4.NF.3 a	Understand a fraction a/b with a > 1 as a sum of fractions 1/b. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.	• Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.	MP 2,7,8
4.NF.3 b-1	Understand a fraction a/b with a > 1 as a sum of fractions 1/b. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.	• Only the answer is required. • Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. • Tasks may include fractions that equal whole numbers. Whole numbers are limited to 0 through 5.	MP 7,8
4.NF.3 c	Understand a fraction a/b with a > 1 as a sum of fractions 1/b. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction	• Tasks do not have a context. • Denominators are limited to grade 3 possibilities (2, 3, 4, 6, 8) so as to keep computational difficulty lower.	MP 7
4.NF.3 d	Understand a fraction a/b with a>1 as a sum of fractions 1/b. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	• Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. • Addition and subtraction situations are limited to the dark- or medium-shaded types in Table 2, p. 9 of the OA Progression document; these situations are sampled equally. • Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy.	MP 1,4,5
4.NF.4 a	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 x (1/4), recording the conclusion by the equation 5/4 = 5 x (1/4)	• Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.	MP 5,7

4.NF.4 b-1	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. b. Understand a multiple of a/b as a multiple of 1/b. For example, use a visual fraction model to express 3 x (2/5) as 6 x (1/5)	• Tasks do not have a context. • Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy. • Results may equal fractions greater than 1 (including fractions equal to whole numbers limited to 0 through 5). • Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.	MP 5,7
4.NF.4 b-2	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. b. Use the understanding that a multiple of a/b is a multiple of 1/b to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 x (2/5) as 6/5. (In general, n x (a/b) = (n x a)/b.)	• Tasks do not have a context. • Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy. • Tasks involve expressing a/b as a multiple of a/b as a fraction. • Results may equal fractions greater than 1 (including fractions equal to whole numbers limited to 0 through 5). • Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.	MP 5,7
4.NF.4 c	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?	• Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy. • Situations are limited to those in which the product is unknown (situations do not include unknown factors). • Situations involve a whole number of fractional quantities—not a fraction of a whole-number quantity. • Results may equal fractions greater than 1 (including fractions equal to whole numbers). • Results may equal fractions greater than 1 (including fractions equal to whole numbers limited to 0 through 5). • Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.	MP 1,4,5

Number Talks

What does Number Talks look like?

- Students are near each other so they can communicate with each other (central meeting place)
- Students are mentally solving problems
- Students are given thinking time
- Thumbs up show when they are ready
- Teacher is recording students' thinking

Communication

- Having to talk out loud about a problem helps students clarify their own thinking
- Allow students to listen to other's strategies and value other's thinking
- Gives the teacher the opportunity to hear student's thinking

Mental Math

- When you are solving a problem mentally you must rely on what you know and understand about the numbers instead of memorized procedures
- You must be efficient when computing mentally because you can hold a lot of quantities in your head

Thumbs Up

- This is just a signal to let you know that you have given your students enough time to think about the problem
- If will give you a picture of who is able to compute mentally and who is struggling
- It isn't as distracting as a waving hand

Teacher as Recorder

- Allows you to record students' thinking in the correct notation
- Provides a visual to look at and refer back to
- Allows you to keep a record of the problems posed and which students offered specific strategies

Purposeful Problems

- Start with small numbers so the students can learn to focus on the strategies instead of getting lost in the numbers
- Use a number string (a string of problems that are related to and scaffold each other)

Starting Number Talks in your Classroom

- Start with specific problems in mind
- Be prepared to offer a strategy from a previous student
- It is ok to put a student's strategy on the backburner
- Limit your number talks to about 15 minutes
- Ask a question, don't tell!

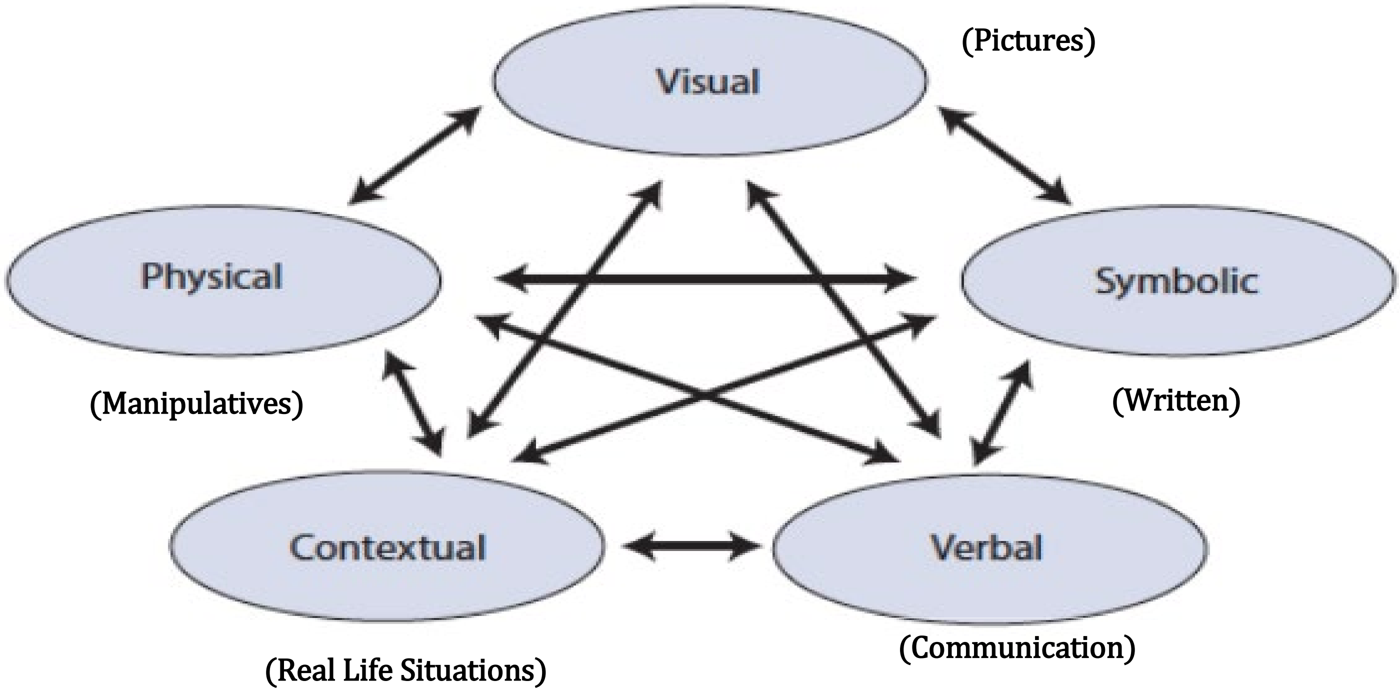
The teacher asks questions:

- Who would like to share their thinking?
- Who did it another way?
- How many people solved it the same way as Billy?
- Does anyone have any questions for Billy?
- Billy, can you tell us where you got that 5?
- How did you figure that out?

Student Name: ____________________ Task: ____________________ School: _________ Teacher: ___________ Date: _________

"I CAN....."	STUDENT FRIENDLY RUBRIC				SCORE
	...a start 1	...getting there 2	...that's it 3	WOW! 4	
Understand	I need help.	I need some help.	I do not need help.	I can help a class-mate.	
Solve	I am unable to use a strategy.	I can start to use a strategy.	I can solve it more than one way.	I can use more than one strategy and talk about how they get to the same answer.	
Say or Write	I am unable to say or write.	I can write or say some of what I did.	I can write and talk about what I did. I can write or talk about why I did it.	I can write and say what I did and why I did it.	
Draw or Show	I am not able to draw or show my thinking.	I can draw, but not show my thinking; or I can show but not draw my thinking;	I can draw and show my thinking	I can draw, show and talk about my think-ing.	

Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

Concrete: “Doing Stage”: Physical manipulation of objects to solve math problems.

Pictorial: “Seeing Stage”: Use of imaged to represent objects when solving math problems.

Abstract: “Symbolic Stage”: Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over…. And then read it again.

DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

WRITE your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

Teacher Questioning:

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple “yes” or “no,” or do they invite students to deepen their understanding?

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.

100 questions that promote Mathematical Discourse

Help students **work together** to make sense of mathematics

1. What **strategy** did you use?
2. Do you **agree**?
3. Do you **disagree**?
4. Would you **ask the rest of the class** that question?
5. Could you **share your method** with the class?
6. What part of what he said **do you understand**?
7. Would someone like to **share** ___?
8. Can you **convince the rest of us** that your answer makes sense?
9. **What do others think** about what [student] said?
10. Can someone **retell or restate** [student]'s explanation?
11. Did you **work together**? In what way?
12. Would anyone like to **add to what was said**?
13. Have you **discussed** this with your group? With others?
14. Did anyone get a **different answer**?
15. **Where** would you go for **help**?
16. **Did everybody get a fair chance** to talk, use the manipulatives, or be the recorder?
17. How could you help another student **without telling them the answer**?
18. **How would you explain** ___ to someone who missed class today?

Help students **rely more on themselves** to determine whether something is mathematically correct

19. Is this a **reasonable answer**?
20. Does that make **sense**?
21. **Why** do you think that? Why is that true?
22. Can you **draw a picture or make a model** to show that?
23. **How** did you reach that conclusion?
24. Does anyone want to **revise** his or her answer?
25. **How were you sure** your answer was right?

Ready

Help students learn to **reason mathematically**

26 **How did you begin** to think about this problem?

27 What is **another way** you could solve this problem?

28 How could you **prove** ______?

29 Can you **explain how your answer is different from or the same as** [student]'s answer?

30 Let's **break the problem into parts**. What would the parts be?

31 Can you **explain this part more specifically**?

32 Does that **always work**?

33 Can you think of a case where that **wouldn't work**?

34 How did you **organize** your information? Your thinking?

Help students **evaluate their own processes** and engage in productive peer interaction

35 What do you need to do **next**?

36 What have you **accomplished**?

37 What are your **strengths and weaknesses**?

38 Was your **group participation appropriate and helpful**?

Help students with **problem comprehension**

39 What is this problem about? What can you **tell me about it**?

40 Do you need to **define or set limits** for the problem?

41 How would you **interpret** that?

42 Could you **reword that in simpler terms**?

43 Is there something that can be **eliminated** or that is **missing**?

44 Could you **explain** what the problem is asking?

45 What **assumptions** do you have to make?

46 What do you **know** about this part?

47 Which words were **most important**? Why?

Help students learn to **conjecture, invent, and solve** problems

48 **What would happen if ___?**

49 Do you see a **pattern**?

50 What are some **possibilities** here?

51 Where could you find the **information** you need?

52 How would you **check your steps** or your answer?

53 What **did not work**?

54 How is your solution method the **same as or different from** [student]'s method?

55 Other than retracing your steps, **how can you determine** if your answers are appropriate?

56 How did you **organize** the information? Do you have a **record**?

57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?

58 What have you tried? What **steps** did you take?

59 How would it look if you used this **model** or these **materials**?

60 How would you draw a **diagram or make a sketch** to solve the problem?

61 Is there **another possible answer**? If so, explain.

62 Is there **another way to solve** the problem?

63 Is there **another model** you could use to solve the problem?

64 Is there anything you've **overlooked**?

65 **How did you think** about the problem?

66 What was your **estimate or prediction**?

67 How **confident** are you in your answer?

68 **What else** would you like to know?

69 What do you think comes **next**?

70 Is the solution **reasonable**, considering the context?

71 Did you have a **system**? Explain it.

72 Did you have a **strategy**? Explain it.

73 Did you have a **design**? Explain it.

Help students learn to **connect mathematics, its ideas, and its application**

74 What is the **relationship** between ___ and ___?

75 Have we ever solved a problem **like this before**?

76 What uses of mathematics did you find in the **newspaper** last night?

77 What is the **same**?

78 What is **different**?

79 Did you use skills or build on concepts that were **not necessarily mathematical**?

80 Which **skills or concepts** did you use?

81 What **ideas** have we explored before that were useful in solving this problem?

82 Is there a **pattern**?

83 **Where else** would this strategy be useful?

84 How does this **relate** to ___?

85 Is there a **general rule**?

86 Is there a **real-life situation** where this could be used?

87 How would your method work with **other problems**?

88 What other problem does this seem to **lead to**?

Help students **persevere**

89 Have you tried making a **guess**?

90 **What else** have you tried?

91 Would **another method** work as well or better?

92 Is there **another way** to draw, explain, or say that?

93 Give me another **related problem**. Is there an easier problem?

94 How would you **explain** what you know right now?

Help students focus on the mathematics from **activities**

95 What was **one thing you learned** (or two, or more)?

96 Did you **notice any patterns**? If so, describe them.

97 What **mathematics topics** were used in this investigation?

98 What were the **mathematical ideas** in this problem?

99 What is mathematically **different about these two situations**?

100 What are the **variables** in this problem? What stays **constant**?

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

3-5 Math Fact Fluency Expectation

3.OA.C.7: Single-digit products and quotients (Products from memory by end of Grade 3)
3.NBT.A.2: Add/subtract within 1000
4.NBT.B.4: Add/subtract within 1,000,000/ Use of Standard Algorithm
5.NBT.B.5: Multi-digit multiplication/ Use of Standard Algorithm

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

Mathematical Proficiency

To be mathematically proficient, a student must have:

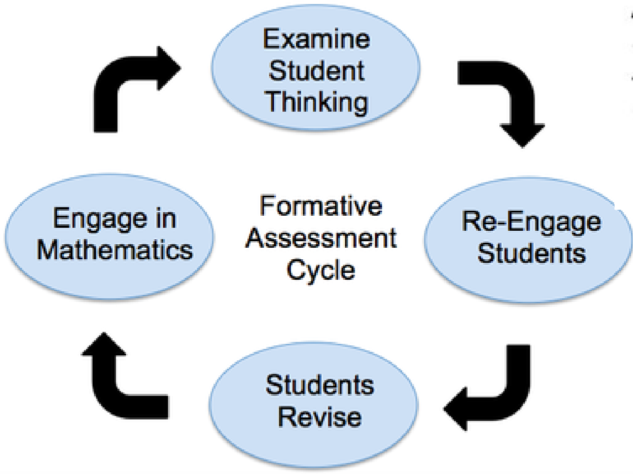
- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.

Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(Wiliam 2007, pp. 1054; 1091)



Connections to the Mathematical Practices

Student Friendly Connections to the Mathematical Practices

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

The Standards for Mathematical Practice:

Describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

1	**Make sense of problems and persevere in solving them** Mathematically proficient students in grade 4 know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
2	**Reason abstractly and quantitatively** Mathematically proficient fourth graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.
3	**Construct viable arguments and critique the reasoning of others** In fourth grade mathematically proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking.
4	**Model with mathematics** Mathematically proficient fourth grade students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evalu-

	ate their results in the context of the situation and reflect on whether the results make sense.
5	**Use appropriate tools strategically** Mathematically proficient fourth graders consider the available tools(including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.
6	**Attend to precision** As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.
7	**Look for and make use of structure** In fourth grade mathematically proficient students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.
8	**Look for and express regularity in repeated reasoning** Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

Effective Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Practices for Orchestrating Productive Mathematics Discussions	
Practice	Description/ Questions
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task? How do you respond to the work that students are likely to produce? Which strategies from student work will be most useful in addressing the mathematical goals?
2. Monitoring	Paying attention to what and how students are thinking during the lesson. Students working in pairs or groups Listening to and making note of what students are discussing and the strategies they are using Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)
3. Selecting	This is the process of deciding the *what* and the *who* to focus on during the discussion.
4. Sequencing	What order will the solutions be shared with the class?
5. Connecting	Asking the questions that will make the mathematics explicit and understandable. Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.

MATH CENTERS/ WORKSTATIONS

Math workstations allow students to engage in authentic and meaningful hands-on learning. They often last for several weeks, giving students time to reinforce or extend their prior instruction. Before students have an opportunity to use the materials in a station, introduce them to the whole class, several times. Once they have an understanding of the concept, the materials are then added to the work stations.

Station Organization and Management Sample

Teacher A has 12 containers labeled 1 to 12. The numbers correspond to the numbers on the rotation chart. She pairs students who can work well together, who have similar skills, and who need more practice on the same concepts or skills. Each day during math work stations, students use the center chart to see which box they will be using and who their partner will be. Everything they need for their station will be in their box. **Each station is differentiated**. If students need more practice and experience working on numbers 0 to 10, those will be the only numbers in their box. If they are ready to move on into the teens, then she will place higher number activities into the box for them to work with.

In the beginning there is a lot of prepping involved in gathering, creating, and organizing the work stations. However, once all of the initial work is complete, the stations are easy to manage. Many of her stations stay in rotation for three or four weeks to give students ample opportunity to master the skills and concepts.

Read *Math Work Stations* by Debbie Diller.
In her book, she leads you step-by-step through the process of implementing work stations.

MATH WORKSTATION INFORMATION CARD

Math Workstation: ____ Time: ____________

NJSLS.:

____

____

Objective(s): By the end of this task, I will be able to:

- ____
- ____
- ____

Task(s):

- ____
- ____
- ____
- ____

Exit Ticket:

- ____
- ____
- ____

MATH WORKSTATION SCHEDULE **Week of: ________________**

DAY	Technology Lab	Problem Solving Lab	Fluency Lab	Math Journal	Small Group Instruction
Mon.	Group ____	Group ____	Group ____	Group ____	BASED ON CURRENT OBSERVATIONAL DATA
Tues.	Group ____	Group ____	Group ____	Group ____	
Wed.	Group ____	Group ____	Group ____	Group ____	
Thurs.	Group ____	Group ____	Group ____	Group ____	
Fri.	Group ____	Group ____	Group ____	Group ____	

INSTRUCTIONAL GROUPING

	GROUP A		GROUP B
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
	GROUP C		GROUP D
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	

Second Grade PLD Rubric

Got It Evidence shows that the student essentially has the target concept or big math idea.		**Not There Yet** Student shows evidence of a major misunderstanding, incorrect concepts or procedure, or a failure to engage in the task.		
PLD Level 5: 100% **Distinguished command**	**PLD Level 4: 89%** **Strong Command**	**PLD Level 3: 79%** **Moderate Command**	**PLD Level 2: 69%** **Partial Command**	**PLD Level 1: 59%** **Little Command**
Student work shows **distinguished levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes an **efficient and logical progression** of mathematical reasoning and understanding.	Student work shows **strong levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes a **logical progression** of mathematical reasoning and understanding.	Student work shows **moderate levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes a **logical but incomplete progression** of mathematical reasoning and understanding. Contains **minor errors.**	Student work shows **partial understanding** of the mathematics. Student **constructs** and **communicates** an **incomplete response** based on student's attempts of explanations/ reasoning using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes an **incomplete or illogical progression** of mathematical reasoning and understanding.	Student work shows **little understanding** of the mathematics. Student **attempts** to **constructs** and **communicates** a response using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes **limited evidence of the progression** of mathematical reasoning and understanding.
5 points	**4 points**	**3 points**	**2 points**	**1 point**

DATA DRIVEN INSTRUCTION

Formative assessments inform instructional decisions. Taking inventories and assessments, observing reading and writing behaviors, studying work samples and listening to student talk are essential components of gathering data. When we take notes, ask questions in a student conference, lean in while a student is working or utilize a more formal assessment we are gathering data. Learning how to take the data and record it in a meaningful way is the beginning of the cycle.

Analysis of the data is an important step in the process. What is this data telling us? We must look for patterns, as well as compare the notes we have taken with work samples and other assessments. We need to decide what are the strengths and needs of individuals, small groups of students and the entire class. Sometimes it helps to work with others at your grade level to analyze the data.

Once we have analyzed our data and created our findings, it is time to make informed instructional decisions. These decisions are guided by the following questions:

- What mathematical practice(s) and strategies will I utilize to teach to these needs?
- What sort of grouping will allow for the best opportunity for the students to learn what it is I see as a need?
- Will I teach these strategies to the whole class, in a small guided group or in an individual conference?
- Which method and grouping will be the most effective and efficient? What specific objective(s) will I be teaching?

Answering these questions will help inform instructional decisions and will influence lesson planning.

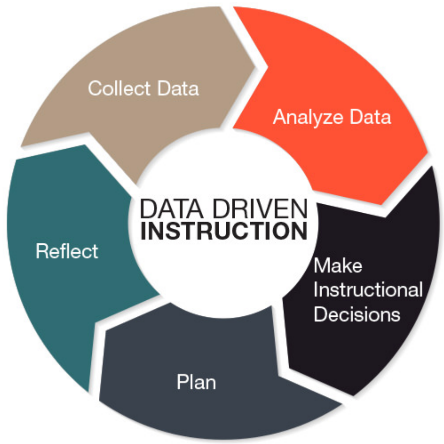
Then we create our instructional plan for the unit/month/week/day and specific lessons.

It's important now to reflect on what you have taught.

Did you observe evidence of student learning through your checks for understanding, and through direct application in student work?

What did you hear and see students doing in their reading and writing?

Now it is time to begin the analysis again.



Data Analysis Form School: ________________ Teacher: ________________________ Date: _____________
Assessment: ____ NJSLS: ____

GROUPS (STUDENT INITIALS)	SUPPORT PLAN	PROGRESS
MASTERED (86% - 100%) (PLD 4/5):		
DEVELOPING (67% - 85%) (PLD 3):		
INSECURE (51%-65%) (PLD 2):		
BEGINNING (0%-50%) (PLD 1):		

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MATH PORTFOLIO EXPECTATIONS

The Student Assessment Portfolios for Mathematics are used as a means of documenting and evaluating students' academic growth and development over time and in relation to the CCSS-M. The September task entry(-ies) should reflect the prior year content and *can serve* as an additional baseline measure.

All tasks contained within the **Student Assessment Portfolios** should be aligned to NJSLS and be "practice forward" (closely aligned to the Standards for Mathematical Practice).

Four (4) or more additional tasks will be included in the **Student Assessment Portfolios** for Student Reflection and will be labeled as such.

K-2 GENERAL PORTFOLIO EXPECTATIONS:

- Tasks contained within the Student Assessment Portfolios are "practice forward" and denoted as "Individual", "Partner/Group", and "Individual w/Opportunity for Student Interviews[1].
- Each Student Assessment Portfolio should contain a "Task Log" that documents all tasks, standards, and rubric scores aligned to the performance level descriptors (PLDs).
- Student work should be attached to a completed rubric; with appropriate teacher feedback on student work.
- Students will have multiple opportunities to revisit certain standards. Teachers will capture each additional opportunity "as a new and separate score" in the task log.
- A 2-pocket folder for each Student Assessment Portfolio is *recommended.*
- All Student Assessment Portfolio entries should be scored and recorded as an Authentic Assessment grade (25%)[2].
- All Student Assessment Portfolios must be clearly labeled, maintained for all students, inclusive of constructive teacher and student feedback and accessible for review.

GRADES K-2

Student Portfolio Review

Provide students the opportunity to review and evaluate their portfolio at various points throughout the year; celebrating their progress and possibly setting goals for future growth. During this process, students <u>should retain ALL of their current artifacts</u> in their Mathematics Portfolio

4^{TH} Grade Authentic Performance Task: Boxing Up Leftover Brownies

Amaria has brownies at her birthday party. At the end of the party there are the following brownies left over:

- 5 brownies with cream cheese frosting
- 4 plain chocolate brownies
- 3 chocolate brownies with nuts
- 7 brownies with caramel frosting

Part 1:
After the party the brownies are put into boxes. A box can hold 8 brownies. If each type of brownie were packed into their own box, what fraction of a box does each type of brownie take up? Draw pictures below to show your work.

Part 2:
Amaria and her Mom want to use fewer boxes and put different types of brownies into the same box. How many whole boxes do they fill? Will there be a box partially filled? If so what fraction of the box is partially filled? Draw pictures to show your work.

Part 3:
Write an equation to match the picture that you drew in Part 2.

Part 4:
Is there space for any more brownies? If so how many more brownies do you have room for? Write an equation that shows your work.

Boxing Up Leftover Brownies 4.NF.3	
Domain	**Number and Operations - Fractions**
Cluster	**Build fractions from unit fractions.**
Standard(s)	**4.NF.3** Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. **4.NF.3.** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. **4.NF.3d** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
Materials	Activity sheet

Rubric		
Level I	**Level II**	**Level III**
Limited Performance • Solutions include many errors and show limited understanding.	Not Yet Proficient • Solutions will include between 1 to 3 errors in various parts of the task.	Proficient in Performance • Solutions include correct answers and show a deep understanding of concepts. • Answers: Part 1: Pictures are correctly drawn and fractions are correctly labeled. Cream cheese: 5/8. Plain: 4/8. Nuts: 3/8. Caramel: 7/8. • Part 2: Picture is correctly drawn. Answer is 2 and 3/8. • Part 3: 5/8 + 4/8 + 3/8 + 7/8 = 2 and 3/8. • Part 4: There is space for 5 more brownies or there is 5/8 of a box empty. Equation: 3 – 2 and 3/8 = 5/8.

Resources

Engage NY
http://www.engageny.org/video-library?f[0]=im_field_subject%3A19

Common Core Tools
http://commoncoretools.me/
http://www.ccsstoolbox.com/
http://www.achievethecore.org/steal-these-tools

Achieve the Core
http://achievethecore.org/dashboard/300/search/6/1/0/1/2/3/4/5/6/7/8/9/10/11/12

Manipulatives
http://nlvm.usu.edu/en/nav/vlibrary.html

http://www.explorelearning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000

http://www.thinkingblocks.com/

Illustrative Math Project :http://illustrativemathematics.org/standards/k8

Inside Mathematics: http://www.insidemathematics.org/index.php/tools-for-teachers

Sample Balance Math Tasks: http://www.nottingham.ac.uk/~ttzedweb/MARS/tasks/

Georgia Department of Education:https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx

Gates Foundations Tasks:http://www.gatesfoundation.org/college-ready-education/Documents/supporting-instruction-cards-math.pdf

Minnesota STEM Teachers' Center:
http://www.scimathmn.org/stemtc/frameworks/721-proportional-relationships

Singapore Math Tests K-12: http://www.misskoh.com

Mobymax.com: http://www.mobymax.com

21st Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .

References

"Eureka Math" *Great Minds.* 2018 < https://greatminds.org/account/products>